

Exercise 12

Find the limit.

$$\lim_{x \rightarrow 3} \frac{\sqrt{x+6} - x}{x^3 - 3x^2}$$

Solution

Plugging in 3 right away for x gives 0 in the denominator, so simplify the function first or rewrite the limit.

$$\begin{aligned} \lim_{x \rightarrow 3} \frac{\sqrt{x+6} - x}{x^3 - 3x^2} &= \lim_{x \rightarrow 3} \frac{\sqrt{x+6} - x}{x^2(x-3)} \\ &= \lim_{x \rightarrow 3} \frac{\sqrt{x+6} - x}{x^2(x-3)} \cdot \frac{\sqrt{x+6} + x}{\sqrt{x+6} + x} \\ &= \lim_{x \rightarrow 3} \frac{(x+6) - x^2}{x^2(x-3)(\sqrt{x+6} + x)} \\ &= \lim_{x \rightarrow 3} \frac{-x^2 + x + 6}{x^2(x-3)(\sqrt{x+6} + x)} \\ &= - \lim_{x \rightarrow 3} \frac{x^2 - x - 6}{x^2(x-3)(\sqrt{x+6} + x)} \\ &= - \lim_{x \rightarrow 3} \frac{(x-3)(x+2)}{x^2(x-3)(\sqrt{x+6} + x)} \\ &= - \lim_{x \rightarrow 3} \frac{x+2}{x^2(\sqrt{x+6} + x)} \\ &= - \frac{3+2}{3^2(\sqrt{3+6} + 3)} \\ &= - \frac{5}{9(6)} \\ &= - \frac{5}{54} \end{aligned}$$